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X-RAY ASTRONOMY, I

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During the summer of 1970, a graduate level course in x-ray astronomy (Physics 248) was offered at the University of Maryland. Four guest lecturers participated in the presentation of a comprehensive survey of x-ray astronomical theory, data, observational techniques and related astrophysical phenomena. In chronological order, the lecturers were S. S. Holt, P. J. Serlemitsos, Y. Pal and E. A. Boldt. These notes represent the material covered in the first quarter of the course.

INTRODUCTION

Less than a decade has passed since the detection of high-energy photons of extra-solar origin. During this time, considerable evidence has been amassed to the effect that observable x-ray emission may be associated with scale sizes ranging from neutron stars to the whole of the intergalactic medium. These notes will attempt to review the experimental and theoretical justification for as many aspects of this new astronomical channel as the authors feel competent to evaluate.

We define the x-ray band of the electromagnetic spectrum rather arbitrarily from about 0.1 keV to some few hundred keV. All photons in this energy window are thus classified as x-rays regardless of the mechanism of their origin. With the upper end of this window below the electron rest mass, we are reasonably certain that nuclear processes play no substantial role in the genesis of celestial x-rays.

CONTINUUM X-RAY PRODUCTION AND ELECTRON ENERGY LOSS

Most of the possible mecha isms for x-ray production, and the ones most likely to be responsible for the greater fraction of the celestial x-ray emission, involve the interaction of a single electron with an electromagnetic field. In such an interaction, the subsequent loss of electron kinetic energy results in the production of an x-ray photon. A detailed presentation of such continuum x-ray production is given in Blumenthal and Gould.[1] The present treatment is considerably simplified, and is based largely upon the notes of Boldt.[2]

The first simplification to the general theory we shall adopt is that we shall concern ourselves with Thomson scattering, alone. Thomson scattering is the interaction of charged particles with electromagnetic radiation in the regine where the unscattered photon energy, $h\nu$, is considerably less than the rest mass of the scattering particle:

$$h\nu << mc^2 \tag{1}$$

For this condition, the cross section, σ_o , is independent of energy and is given by[3]

$$\sigma_o = \frac{8}{3}\pi r^2 = 7\times10^{-25}\ \mathrm{cm}^2 \tag{2}$$

where r is the classical electron radius

$$r = \frac{e^2}{mc^2} = 3\times10^{-13}\ \mathrm{cm} \tag{3}$$

The x-ray photons themselves satisfy the Thomson condition, and, as these photons will generally be produced from lower energy photons, we need not concern ourselves in detail with x-ray production from gamma rays where the Thomson cross-section gives way to the energy-dependent Klein-Nishina cross-section.

In the Thomson limit, the mean kinetic energy loss rate of a free electron may be written

$$-\frac{dE}{dt} = \sigma_0 v \frac{\rho}{\epsilon} \langle h\nu \rangle \tag{4}$$

where E is the electron kinetic energy measured in the frame where the time is t and the electron velocity is v, ρ and ϵ are the energy density and mean quantum energy of the target electromagnetic field, and $\langle h\nu \rangle$ is the mean energy of the emitted photon. The exact expression for the classical radiation loss of an electron in an electromagnetic field is:[4]

$$-\frac{dE}{dt} = \frac{2}{3} r^2 \left[\left(\vec{E} + \frac{\vec{v}}{c} \times \vec{H} \right)^2 - \frac{1}{c^2} \left(\vec{E} \cdot \vec{v} \right)^2 \right] \gamma^2 c \tag{5}$$

where $\vec{E}$ and $\vec{H}$ are the electric and magnetic components of the target field, c is the velocity of light and γ is the electron Lorentz factor. The identity of equations (4) and (5) under a variety of simple assumptions about the target field may be used to infer specific results which will be shown to be characteristic of situations which we are reasonably certain obtain in x-ray astronomy.

If we consider equation (5) in the limit of plane waves, i.e.

$$\vec{n} \cdot \vec{E} = 0 \tag{6}$$

$$\vec{H} = \vec{n} \times \vec{E} \tag{7}$$

and further assume a randomly oriented photon flux

$$\langle \vec{n} \cdot \vec{v} \rangle = 0 \tag{8}$$

we can study the production of x-rays via the Compton mechanism (in the literature it is often referred to as the inverse Compton process, as the electron loses rather than gains energy, but the physics is the same in either case). In such elastic collisions of free electrons with photons, equation (5) becomes

$$-\frac{dE}{dt} = \sigma_0 c \, \frac{(H^2+E^2)}{8\pi}\left(1+\frac{\beta^2}{3}\right)\gamma^2 = 2\times10^{-14}\left(1+\frac{\beta^2}{3}\right)\gamma^2 \rho \tag{9}$$

which, in the limit of relativistic electrons, prescribes the mean emitted photon energy

$$\langle h\nu \rangle = \frac{4}{3}\gamma^2 \epsilon \tag{10}$$

This is precisely that limit in which Compton scattering is most important in x-ray astronomy, i.e., the Thomson scattering of ultra-relativistic electrons on starlight, infrared or microwave photons.

We obtain a similar expression for the case of the interaction of electrons with a pure magnetic field (called synchrotron emission in the bulk of the literature, although the phrase magnetic bremsstrahlung is also used). If we assume a random field distribution

$$H_\perp^2 = \frac{\langle |\vec{v}\times\vec{H}|^2 \rangle}{v^2} = \frac{2}{3}H^2 \tag{11}$$

where the factor $\frac{2}{3}$ arises from integration over all solid angles. The resultant energy loss is

$$-\frac{dE}{dt} = \frac{4}{9} r^2 c \beta^2 \gamma^2 H^2 \approx 10^{-15} \beta^2 \gamma^2 H^2 \tag{12}$$

which, in the ultra-relativistic limit, results in an expression for the mean emitted photon energy which is identical with that for Compton scattering:

$$\langle h\nu \rangle = \frac{4}{3} \gamma^2 \epsilon \tag{13}$$

In this case, the meaning of ϵ is a bit more obscure than in the Compton case, but it can be formally evaluated from synchrotron theory. Non-relativistically, the emission is solely at the cyclotron frequency

$$\langle h\nu \rangle = \frac{he}{2\pi mc} \frac{H_\perp}{\gamma} \tag{14}$$

As the electron energy increases, higher harmonics are generated until an effective continuum is observed, with a maximum at an energy[5]

$$\langle h\nu \rangle = .44 \frac{he}{2\pi mc} H_\perp \gamma^2 = 8 \times 10^{-21} H_\perp \gamma^2 \tag{15}$$

Again, the ultra-relativistic limit is the most interesting, wherein x-rays may be produced in relatively weak ambient magnetic fields.

Finally, we consider a pure electric field, for which equation (5) becomes

$$-\frac{dE}{dt} = 2\sigma_0 c \rho \left(1 - \frac{\beta^2}{3}\right) \gamma^2 \tag{16}$$

for the randomly oriented case, i.e., for

$$\langle \cos^2 \Theta \rangle = \frac{1}{3} \tag{17}$$

The energy density in the target electric field is assumed to arise from a number density n_0 of discrete charges Ze which may be localized within an impact parameter

$$b = \frac{\lambda}{4} = \frac{h}{4p} \tag{18}$$

This energy density is then

$$\rho = n_0 \left(\int_b^{\infty} \frac{(Ze/r^2)^2}{8\pi} 4\pi r^2 dr \right) \tag{19}$$

which, in the non-relativistic limit, yields the expression:

$$-\frac{dE}{dt} = \frac{16}{3} \alpha Z^2 r^2 (mc^2) \beta c n_0 \approx 8.6\times10^{-23} Z^2 \beta n_0 \tag{20}$$

The above expression is, in fact, the correct non-relativistic expression and, since the conventional approximation to $\langle h\nu \rangle$ is

$$\langle h\nu \rangle \approx \frac{E}{2} \tag{21}$$

it would appear that the non-relativistic limit is the one which is appropriate to x-ray astronomy. It should be remembered that bremsstrahlung differs in at least two important respects from Compton and synchrotron emission. The non-relativistic nature of the source electrons is the obvious difference, but the relatively low yield is another important consideration, i.e., the energy loss in radiated x-ray photons via equation (20) constitutes only ~ 10^{-4} of the total energy loss of such electrons in Coulomb collisions, while the radiation yield in Compton and synchrotron processes is perfect.

Finally, we should consider the bremsstrahlung loss of relativistic electrons not because we expect the x-ray emission from this process to be important (although the radiation yield is significantly greater at these energies), but primarily because the energy loss rate of relativistic electrons may be dominated by non-radiative Coulomb collisions for some astrophysical situations in which we might expect Compton or synchrotron emission to be important. The total electron energy loss may be written[5,6] as a sum of radiative and non-radiative terms. The non-radiative loss in cold (non-ionized) hydrogen is

$$-\frac{dE}{dt} = 1.2\times10^{-20}\, n_0 \left\{3 \ln\gamma + 18.8\right\} \tag{22}$$

The radiation loss for cold hydrogen is

$$-\frac{dE}{dt} = 8\times10^{-16}\, E\, n_0 \tag{23}$$

and is decreased by ~ 10% for a cold target gas of universal abundance. For an ionized medium, equation (22) is replaced by

$$-\frac{dE}{dt} = 1.2\times10^{-20}\, n_0 \left\{\ln\gamma - \ln n_0 + 73.4\right\} \tag{24}$$

and equation (23) by

$$-\frac{dE}{dt} = 1.37\times10^{-16}\, n_0 E \left\{\ln\gamma + .36\right\} \tag{25}$$

As this section has been treated entirely within the framework of the energy loss of electrons, we conclude by defining a characteristic energy loss time τ:

$$\tau \equiv \left(\frac{1}{E}\frac{dE}{dt}\right)^{-1} \tag{26}$$

which we shall subsequently find quite useful in the determination of the physical conditions required for source models.

SPECTRA

In the case of Compton and synchrotron emission, the photon spectra at each electron energy are sharply peaked, justifying the use of a δ-function approximation:

$$\frac{d}{dh\nu}\left(\frac{dE}{dt}\right) = \frac{dE}{dt}\left\{\delta\left(h\nu - \langle h\nu\rangle\right)\right\} \tag{27}$$

In the case of non-relativistic bremsstrahlung, the radiation spectrum does not exhibit the same sort of relatively sharp maximum:

$$\frac{d}{dh\nu}\left(\frac{dE}{dt}\right) \approx \frac{1}{E}\,\frac{dE}{dt} \tag{28}$$

The overall photon spectra generated from a given electron spectrum may then be evaluated via

$$\frac{dq}{dh\nu}\left(\frac{\text{photons}}{\text{cm}^3\text{-erg-sec}}\right) = \frac{1}{h\nu}\int_{h\nu}^{\infty}\left[\frac{d}{dh\nu}\left(\frac{dE}{dt}\right)\right]\left[\frac{d}{dE}\left(\frac{dN}{dV}\right)\right]dE \tag{29}$$

so that the insertion of an electron energy spectrum into equation (29) immediately gives an approximation to the x-ray spectrum expected from each of the continuum processes. For example, we expect that almost any non-equilibrium electron distribution can be represented, at least over a limited energy range, by a power law:

$$\frac{d}{dE}\left(\frac{dN}{dV}\right) \propto E^{-\Gamma}\left(\frac{1}{\beta}\right) \tag{30}$$

(where the $1/\beta$ is inserted to simplify the spectral form obtained from non-relativistic bremsstrahlung; for the other two processes, of course, the factor $1/\beta$ is unity). We then obtain the spectral forms:

Bremsstrahlung: $$\frac{dq}{dh\nu} \propto (h\nu)^{-(\Gamma+1)} \tag{31}$$

Compton: $$\frac{dq}{d h\nu} \propto (h\nu)^{-\left(\frac{\Gamma+1}{2}\right)} \tag{32}$$

Synchrotron: $$\frac{dq}{d h\nu} \propto (h\nu)^{-\left(\frac{\Gamma+1}{2}\right)} \tag{33}$$

Note that the Compton and synchrotron output spectra are identical and, as both mechanisms are characteristic of non-equilibrium electron distributions, a power law representation should be generally valid. The stability of the radiated spectra are estimable from the results already obtained, as well. Given an initial electron spectrum of the form of equation (30), the Compton and synchrotron losses predominantly affect the more energetic electrons since

$$-\frac{dE}{dt} \propto \gamma^2 \tag{34}$$

For a spectrum of electrons which are created at one time and allowed to decay via one of these mechanisms, we expect that the spectrum will continually steepen with increasing energy such that above an energy corresponding to the characteristic time τ all electrons will have been effectively dissipated. On the other hand, a continuously created electron spectrum exhibits a very interesting feature. If such creation occurs uniformly after a starting point a time τ in the past, the electron spectrum will be very close to the creation power law up to the energy corresponding to τ, while at higher energies it will be well represented by a power law one unit steeper in index. Remembering that a unit change in electron index will yield an index change of 1/2 in the photon spectrum across the spectral break, such a discontinuity in the slope of observed x-rays will not only be indicative of a non-collisional origin for the

x-rays, but will date the birth of the source consistent with the source parameters and the assumption of uniform electron acceleration throughout the lifetime.

Non-thermal bremsstrahlung exhibits a different temporal behavior, as the energy loss of the electrons is relatively independent of energy. We expect, therefore, that the photon spectrum will decay without altering its shape appreciably within the characteristic decay time. It must be remembered however, that the relevant characteristic decay time is approximately 4 orders of magnitude larger than that for radiation alone, so that thermalization of the electron population is expected to be achieved well before the characteristic radiation time. We expect, then, that any non-fluctuating bremsstrahlung emission will almost certainly be characterized by a Maxwellian electron distribution:

$$\frac{d}{dE}\frac{dN}{dV} \propto E^{1/2} \exp\left(-E/kT\right) \tag{35}$$

so that the free-free component of the source function from a thermal source should be well represented by

$$\frac{dq}{dh\nu} = 1.1\times10^{-11}\,\frac{Z^2 g}{T^{1/2}}\,\frac{n_o n_e}{h\nu}\exp\left(-h\nu/kT\right) \tag{36}$$

where n_e is the electron density in collision with the ion density n_o of charge Ze, and g is the free-free gaunt factor (an energy- and Z-dependent correction to the Born approximation which is, typically, of order unity). In addition to the gaunt factor complicating the output spectrum, line emission and recombination radiation must be added to the emission[7]. Furthermore, the effects of scattering and reabsorption in the source will make the output spectrum deviate considerably from the optically

thin case above, in general. The case of complete optical thickness is, of course, black-body radiation:

$$\frac{dq}{dh\nu} \propto \frac{(h\nu)^2}{\exp(h\nu/kT) - 1} \tag{37}$$

The intermediate case of partial optical thickness is studied via the free-free absorption coefficient:

$$\mathcal{K}_\nu \approx 10^{-73} \frac{z_a^2 n_e n_o}{(kT)^{1/2}(h\nu)^3}\left(1 - e^{-h\nu/kT}\right) \quad \text{cm}^{-1} \tag{38}$$

which is derived from the optically thin emission from equation (36) and the Einstein B coefficient. In the Rayleigh-Jeans limit:

$$\mathcal{K}_\nu \approx 10^{-78} \frac{z_a^2 n_e n_o}{(kT)^{3/2}(h\nu)^2} \quad \text{cm}^{-1} \tag{39}$$

We must also take Thomson scattering of the x-rays on electrons in the source into account[8]. Thomson scattering will smooth out the spectrum, as the speed of electrons in a source where kT is of the order of several kilovolts is ~ 0.1 c, so that there will be substantial Doppler spreading of the output spectrum. In addition, Thomson scattering will increase the path in the source for all photons, thereby increasing the probability for free-free absorption. The Thomson scattering coefficient is, of course,

$$\mathcal{K}_T = \sigma_o n_o = 7\times 10^{-25} n_o \quad \text{cm}^{-1} \tag{40}$$

Neutron diffusion theory (applicable where the number of Thomson scatterings is large) gives the effective absorption coefficient; if we assume uniform x-ray production is a sphere of radius R, the dimensionless optical depth for the source is:

$$\lambda_\nu = (3\mathcal{K}_\nu \mathcal{K}_T)^{1/2} R \tag{41}$$

SOURCE REGIMES

From the already calculated energy loss rates and mean radiated energies via the three mechanisms discussed, we can deduce some crude rules of thumb for the viability of each of these processes with respect to specific types of sources. Using a mean radiated photon energy of ~ 10 keV, we can immediately determine the required electron energy for x-ray production for various source parameters. In the case of non-relativistic bremsstrahlung, we always require electrons of comparable energy:

$$E \approx 20\,\mathrm{keV} \approx 3\times10^{-8}\,\mathrm{erg} \quad (\beta \approx .1) \tag{42}$$

The electron energy required for Compton and synchrotron x-rays is strictly a function of the mean target quantum energy:

$$\gamma^2 = \frac{3}{4}\frac{\langle h\nu\rangle}{\epsilon} \tag{43}$$

For the Compton process, the two most important well-established sources of target photons are starlight ($\epsilon \approx 2\,\mathrm{eV}$) and the 3° universal background radiation ($\epsilon = 2.7\,kT = 7\times10^{-4}\,\mathrm{eV}$). For these two instances, equation (43) gives γ^2 of 4×10^3 and 10^7, respectively. The recently discovered (and still controversial) infra-red background would require electrons with energies intermediate to the above two cases, and with ρ ~ 50 times larger than that for the 3° radiation it certainly cannot be ignored if it is real. Note that for all of these possible Compton sources, the Thomson approximation is justified, as is the use of the ultra-relativistic limit.

In the synchrotron case, the ultra-relativistic limit is even better justified. If we assume rough equipartition of starlight, magnetic field

and cosmic ray energy densities in the galaxy so that

$$\rho = \frac{H^2}{8\pi} \approx 1 \ \mathrm{eV/cm^3} \tag{44}$$

we obtain a mean galactic field of $\sim 6\ \mu g$, which we expect is about right to account for galactic radio emission via the synchrotron process. Such a field will require $\chi^2 \approx 4 \times 10^{17}$ for the production of 10 keV x-rays. In a nebula, where a milligauss field might be expected, $\chi^2 \approx 2 \times 10^{15}$; in fact, the ultra-relativistic limit is valid up to fields in excess of 10^{10} gauss. The only case of astrophysical interest in which such a field will be exceeded is at the surface of a neutron star, where the field may be $\sim 10^{12}$ gauss ($\chi^2 \approx 2$).

In order to estimate the relative importance of these processes, we shall assume what we believe to be a "typical" electron spectrum. The existing data in x-ray astronomy is such that those sources which can be well approximated by power law spectra imply electron spectra which are not dissimilar to the cosmic ray spectrum at high energies. We shall, therefore, adopt an electron spectrum for purposes of rough comparison which is of the form:

$$\frac{d}{dE}\frac{dN}{dV} \propto E^{-5/2} \tag{45}$$

The competition between Compton and synchrotron emission must be carefully defined. If we seek the relative importance of the two energy loss mechanisms from the point of view of the energy loss at a particular energy (the usual case in cosmic ray studies), the appropriate parameter is:

$$\left.\frac{dE}{dt}\right|_c \Big/ \left.\frac{dE}{dt}\right|_s = 8\pi\frac{\rho}{H^2} \tag{46}$$

which is valid at all (ultrarelativistic) energies (ρ is the energy density of Compton target photons). In considering such relative importance of energy loss mechanisms, collision losses should also be added when the temporal stability of the electron spectrum is at issue.

For the present, we shall assume that the electron spectrum of equation (45) is temporally stable, so that the comparative energy losses which concern us are not those at constant electron energy, but those at constant emitted photon energy. In other words, the ratio of Compton-produced to synchrotron-produced x-rays is

$$\mathcal{R} = \frac{n_c}{n_s} \frac{8\pi\rho}{H^2} \left(\frac{\gamma_c}{\gamma_s}\right)^2 \tag{47}$$

where the subscripts c and s on the electron densities are at those Lorentz factors suitable for the production of such x-rays by the Compton and synchrotron mechanisms, respectively.

If we consider, for example, x-ray production in the galactic medium via these processes, we know that Compton photons will be produced from both starlight and black-body photons (neglect the controversial infra-red component here). Each of these target media contains ~ 1 eV/cm^3, but at very different photon energies. Their relative importance is easily computed via

$$\frac{\gamma^2 n \text{ (optical)}}{\gamma^2 n \text{ (blackbody)}} \approx 10 \tag{48}$$

so that we expect that starlight photons are responsible for about an order of magnitude more x-rays than are black-body photons in the galactic plane. If the 3° radiation is truly universal, the energy density should not decrease in intergalactic medium (as starlight energy density does),

so that Compton-produced x-rays from the intergalactic medium should primarily arise from black-body photons.

With regard to the competition between synchrotron and Compton interactions in the galaxy, equation (47) yields $R \approx 4 \times 10^3$ in favor of Compton interactions. In the intergalactic medium, where the field is weaker, we would expect a ratio which is still larger. In nebulae, however, where the field may be $\sim 10^{-3}$ gauss, synchrotron emission is doubly enhanced by the increased field energy density and the fact that lower energy (hence, more numerous) electrons are required. Therefore, even though the starlight density may be two orders of magnitude higher than in the interstellar medium, we expect that synchrotron emission may dominate the x-ray emission of nebulae. It must be borne in mind that such arguments should not be considered more than semi-quantitative, as they are predicated upon the assumption of a single power law electron spectrum of index - 2.5 which extends over more than seven orders of magnitude in electron energy.

The lifetime against Compton and synchrotron radiation is independent of the ambient matter density, but the lifetime of the ultra-relativistic electrons responsible for such emission may be collision-dominated if the source is not tenuous enough. Note especially that equation (26) gives a lifetime against synchrotron emission in the nebular case of $\sim$ 1 year. This is particularly disturbing as the emission from the Crab Nebula (which exhibits all of the characteristics of a synchrotron source) has shown no variation in x-ray intensity in the half-dozen years in which such observations have been made. The resolution of this puzzle will be discussed in a later lecture.

With regard to non-relativistic bremsstrahlung, if we naiively extend the spectrum of index -2.5 down from the ultra-relativistic region, an analogue of equation (47) is:

$$R' = \frac{n_c}{n_b} \frac{3\times10^{-14}\,\rho\,\gamma_c^2}{3\times10^{-23}\,\beta\,n_o} \approx 10^6 \qquad (49)$$

in the interstellar medium. The applicability of a single power law extension is certainly questionable, especially at low energies. The lifetime of the electrons is not contraversial, however, being $\sim 10^3$ years. In less tenuous media, this lifetime will scale inversely as the ambient density. For situations where $R' < 1$ (i.e. for $n_o \gtrsim 10^6$ cm^{-3}), we can expect non-relativistic bremsstrahlung to dominate the x-ray emission (e.g. in the solar atmosphere), albeit with characteristic lifetimes much less than a day.

Finally, a few comments about the nature of x-ray observations which have been made to date would seem to be in order. The x-radiation measured from celestial sources is, in general, made with mechanically collimated detectors which integrate the emission over a substantial fraction (usually all) of the source volume. If the source is large enough in angular diameter for a surface brightness measurement to be made, the observed photon flux is:

$$\frac{dI}{dh\nu}\left(\frac{\text{photons}}{\text{cm}^2\text{-sec-sr-erg}}\right) = \frac{1}{4\pi}\int_0^\infty q\,dr \qquad (50)$$

where r is the distance measured from the detector along its line of sight, and q is the local value of the source function which contributes at each point. Since the sources are smaller than the detector field of view in most of the cases in x-ray astronomy, the measured quantity is

usually the scalar flux:

$$\frac{dF}{dh\nu}\left(\frac{\text{photons}}{\text{cm}^2\text{-sec-erg}}\right) = \frac{1}{d^2}\int_0^R g r^2 dr \tag{51}$$

where R is the radius of the source at a distance d from the detector.

From the source functions computed earlier, equation (51) allows us to infer specific source parameters from the measured flux. For example, the assumption of a spherically homogeneous optically thin thermal source enables us, through the source function (36), to compute the volume emission measure from the shape of the spectrum (which defines the temperature T), the absolute scalar flux, and an assumed distance d to the source:

$$n^2 V = \frac{dF}{dh\nu}\,\frac{4\pi d^2 T^{1/2} h\nu}{1.1\times 10^{-11} Z^2 \bar{g}}\, e^{+h\nu/kT} \tag{52}$$

Historically, astrophysical observations have been made with respect to a black-body standard candle, so that it is appropriate to define the "brightness temperature" even for sources which have a nature which is quite different from a black-body. From equation (50), we can define the spectral intensity

$$I_\nu = \left(h\nu\right)^2 \frac{dJ}{dh\nu} = \frac{2h\nu^3}{c^2}\,\frac{1}{e^{h\nu/kT}-1} \tag{53}$$

The brightness temperature is then defined

$$T_B = \frac{h\nu}{k\ln\left(\frac{2h\nu^3}{c^2 I_\nu}+1\right)} \tag{54}$$

In the Rayleigh-Jeans limit, we obtain

$$T_B \approx \frac{I_\nu}{2k}\lambda^2 \tag{55}$$

This limit is generally valid in radio astronomy, but is generally violated in x-ray astronomy. The value of the brightness temperature is that it is a unique parameter which may be operationally defined from a single measurement. Its chief drawbacks in the x-ray case are first, that x-ray sources are not well represented by black-body spectra and, second, that the angular extent of an x-ray source is generally smaller than the detector field of view, so that the surface brightness (hence T_B) cannot be defined, anyway.

THE SUN

The sun emits $\sim 4 \times 10^{33}$ erg/sec, with the greater part of this energy in that portion of the spectrum (including the visible and infra-red) describable by an approximate black-body spectrum at $\sim 6000°K$. For wavelengths longer than ~ 1 cm, the solar emission exceeds the Rayleigh-Jean limit at 6000°K and approaches that of a black-body at $\sim 10^6 °K$. Similarly, the ultra-violet and x-ray spectra are characteristic of an optically thin source at a few million degrees. This high energy emission arises from the hot solar corona (solar densities $\lesssim 10^{10} cm^{-3}$), where the quiescent temperature is $1\text{-}2 \times 10^6 °K$. Any active regions on the sun tend to enhance the x-ray emission considerably.

For large solar flares, several interesting x-ray phenomena have been observed to be characteristic of this type of event. Firstly, the flash phase of the flare seems to be well associated with rapidly rising ($<<$ 1 minute) very hard x-ray emission ($\gtrsim$ 100 keV). There may be several such hard x-ray spikes in the bursts. These bursts are also well correlated in time with type IV bursts at centimeter wavelengths which, since they exhibit considerable polarization, are undoubtedly synchrotron-produced. If we assume a synchrotron origin for these hard x-rays, and we assume a field which may be of the order of 10^3 gauss, we demand electrons considerably higher in energy ($\gamma \sim 10^5$) than have ever been observed to emanate from the sun. The lifetime of such electrons against synchrotron radiation is $\lesssim 10^3$ seconds, however, which is in agreement with the observed decay time of ~ 1 minute. There are several problems with this model, however. First is the fact that none of these electrons are observed to emerge, and it would be reasonable to expect

the higher energy electrons to escape more easily than the lower energy electrons ($\gamma \lesssim 10$) which have been observed directly. Nor is there any other evidence to support the assumption of fields in excess of 10^3 gauss high enough in the solar corona for the synchrotron losses to exceed the collision losses. Most crucial is the fact that the centimeter emission exactly duplicates the x-ray emission, requiring that electrons which are barely relativistic exhibit the same temporal behavior as ultra-relativistic ones.

An approach which has now become fairly well established is to assume that the <u>same</u> electrons are responsible for both emissions, i.e., slightly relativistic electrons which produce x-rays via bremsstrahlung and radio emission via the synchrotron process. This procedure requires the self-absorption of the synchrotron emission at radio wavelengths in the source itself in order that the relative intensity of the two emissions match, but this assumption is justified by the consistency of the whole model and the fact that the differential polarization of the source in time is well explained by this procedure[9].

With regard to the bremsstrahlung, the observed x-ray spectrum is such that the electron spectrum can be well approximated by:

$$\frac{d}{dE}\frac{dN}{dV} \propto E^{-3} \tag{56}$$

The total emission observed then defines the product of electron and ion densities and, since the whole source is observed at once, the volume. Typically, the value at maximum for this emission measure is:

$$n n_0 V \approx 10^{46} \text{ cm}^{-3} \tag{57}$$

The decay time of $\sim$ 1 minute can then be used to define n_o via the energy loss rate for collisions in the non-relativistic regime. This yields a hydrogen density in the range 10^9 - 10^{10} cm^{-3}, and a total number of radiating electrons in the range 10^{36-37}.

As these high energy electrons decay, the source region becomes thermalized, resulting in the type of non-thermal bremsstrahlung from a Maxwellian gas we have already discussed. This source is observed to have a temperature of $\sim 10^7$°K, a decay time of $\sim$ 1 hour, and an emission measure which may be as high as 10^{52} cm^{-3}. This would appear to be consistent with a source which rises higher in the solar corona with time, which would account for the fact that the radio emission moves to longer wavelengths with time because the plasma frequency, hence the low frequency limit to the radiation which can escape, decreases with decreasing density. Because detailed measurements of the spectrum can be made at 1 AU, we can obtain valuable data on ionization equilibria at these temperatures via the emission lines from multiply-ionized elements in the emission spectrum. The free-free continuum as well, can be studied more carefully than the approximate spectrum of equation (36).

X-ray emission from the sun, the only easily available stellar prototype, would then appear to be consistent with the general considerations discussed earlier. With no evidence for anomalously high magnetic fields, the emission has a bremsstrahlung origin: originally non-thermal on a very rapid time scale, followed by more slowly decaying thermal emission complete with the line emission expected from such a source.

SCO X-1

The early measurements of Sco X-1 indicated a spectrum which looked very much like that of thermal bremsstrahlung at a temperature of ~ 50 million degrees. The source is brightest in the sky at ~ 1 keV, yielding about 20 photons $cm^{-2}sec^{-1}$ above this energy. The high apparent luminosity of this source has enabled its location with modulation collimators to better than one arc-minute, leaving only three visible objects within the error box(10). The fact that an optically thin bremsstrahlung spectrum would be relatively flat at energies below the characteristic temperature led observers to expect an optical counterpart which was more blue than usual. One of the three objects in the box was a blue star of about the right magnitude one would expect from extrapolating the x-ray spectrum, and this identification has held up quite well for several years.

At lower frequencies, a thermal source must exhibit some opacity consistent with the Rayleigh-Jeans limit. The onset of such opacity is particularly valuable in the determination of source parameters.

For Sco X-1, for example, if we take $kT \approx 6$ keV we obtain from Equation (39) for $g \approx Z^2 \approx 1$

$$\varkappa_\nu = 10^{-66} \frac{n n_0}{(h\nu)^2} \text{ cm}^{-1} \tag{58}$$

The optical free-free coefficient is then (for $h\nu \approx 2$ eV):

$$\varkappa_\nu \approx 2\times10^{-43}\, n n_0 \tag{59}$$

while at soft x-ray energies ($h\nu \approx 2$ keV):

$$\varkappa_\nu \approx 2\times10^{-49}\, n n_0 \tag{60}$$

Considering the optical depth enhancement from Thomson scattering from equation (41), the optical and soft x-ray optical depths for Sco X-1 are then:

$$\ell\,(2eV) = 6\times10^{-34}\, n^{3/2} R$$
$$\ell\,(2keV) = 6\times10^{-36}\, n^{3/2} R \qquad (61)$$

since the electron and ion densities are essentially the same.

If the output spectrum down to optical wavelengths is calculated carefully, present measurements indicate that the optical depth is of order unity in order to reconcile the x-ray and optical emission. This is after taking interstellar absorption into account if it is assumed that the source is about as close as it can be without proper motion having been observed (d ~ 300 pc). If we adopt $\ell_{opt} \approx 1$, equation (61) gives us

$$n^{3/2} R \approx 10^{33} \qquad (62)$$

Furthermore, the assumption of uniform production gives us, through equation (52), another relation between n and R. Very crudely, the measured spectrum from Sco X-1 is:

$$h\nu \frac{dF}{dh\nu} \approx 10^{2} e^{-h\nu/kT} \ cm^{-2} sec^{-1} \qquad (63)$$

so that

$$n^2 R^3 \approx 10^{17} d^2 \approx 10^{59} \qquad (64)$$

Equations (62 and (64) then demand that:

$$n \approx 10^{16}\ cm^{-3}$$
$$R \approx 10^{9}\ cm \qquad (65)$$

Note that the solar radius is ~ 10^{11} cm, so that the Sco X-1 object is deduced to be quite compact.

We can test the viability of this procedure with additional measurements at other wavelengths. If the source has an optical thickness of unity in the visible, we certainly should expect to see Rayleigh-Jeans behavior in the infra-red and radio. In fact, such infra-red data has been recorded[11]. From that data, we obtain

$$\frac{R^2 T}{d^2} = 10^{-17} \ {}^{\circ}K \qquad (66)$$

which, combined with equation (64), eleminates the distance d:

$$\frac{n^2 R}{T} = 10^{34} \ cm^{-5} \ {}^{\circ}K^{-1} \qquad (67)$$

and gives a consistent picture with the estimates of n and R obtained previously.

We note that the line emission expected from this model should be considerably broadened if, in fact, the emission is made uniformly in the volume (i.e., $\ell_T \gtrsim 7$). Such line emission has been searched for, most particularly from high ionization states of iron, but has not been positively detected as yet (although two groups have achieved positive results of modest statistical significance[12,13]).

We remark, as well, that there are observational parameters which do not easily fit into the framework of this thermal model. Optical line emission which must originate in a region where the temperature is $10^5\,{}^{\circ}K$, and radio emission which is orders of magnitude in excess of the Rayleigh-Jeans extension have been observed. It has been suggested that both of these originate in a cooler medium which is exterior to the x-ray producing volume.

In addition, there is optical flickering in the source with characteristic times ranging between minutes and hours, and substantial hard x-ray emission variations with time scales which are roughly the same[14]. No such rapid variations have been observed in soft x-rays[15]. Qualitatively, this behavior is quite similar to that of a flaring sun, where the soft x-ray emission has a slower temporal behavior than does the hard x-ray emission. It is also essential to note that there must be some fairly rapid source of energy input into such a system, as the cooling of such a source via bremsstrahlung emission should be less than one sec. at this temperature and density. It may be that such hard emission represents some energy input, but it must be remembered that in order to maintain the luminosity of Sco X-1 such input has to be essentially continuous (while the observations indicate that this hard x-ray emission is not always present).

It has been suggested that the energy input may arise from the accretion of matter from a red giant on a companion white dwarf[16]. Although there exists no direct evidence for binary nature in the source, such as periodic Doppler shifting in emission lines, it still remains an attractive possibility. There are several examples of such binary "old novae" in the celestial catalogue, consisting of a red giant and a blue companion. A white dwarf (or smaller) is required for Sco X-1 for two reasons: the deduced extent of the x-ray source, and, more important, the necessary gravitational heating to maintain the source. For example, the gravitational energy converted when a proton falls from infinity onto a unit solar mass (2×10^{33}g) of unit solar radius (7×10^{10}cm) is only ~ 2 keV. Since the proton is being pulled out of a gravitational

well with similar energy if both are ordinary stars, the transfer of energy is much too small. If the accreting object is a neutron star, the problem adds another degree of complexity as any line emission in the source will be gravitationally red-shifted by an amount

$$z = \frac{\nu - \nu_0}{\nu_0} = \left[1 - \frac{GM}{c^2 R^3}\right]^{-1/2} \qquad (68)$$

It would appear, therefore, that a truly definitive model for Sco X-1 must await the next generation of x-ray astronomical measurements, when, hopefully, spectral lines can be measured unambiguously.

CRAB NEBULA

The Crab Nebula was observed at inception (the "accepted" date is July 4, 1054) by Chinese and American Indian observers. The American Indian records consist solely of cave paintings, but the Chinese measurements are remarkably quantitative for that time, enabling modern astronomers to study the light curve (luminosity as a function of time) with at least some measure of quantitative assurance.

The Crab, as a relatively recent close and bright supernova remnant, had been studied in detail prior to the discovery of its x-ray emission. From more than a half century of optical plates, we have evidence that the nebula is still expanding at a rate of $\sim 10^3$ km/sec. In addition, there are filamentary wisps in the nebula which seem to originate at the center of the nebula with a frequency of a few times per year. These wisps rapidly move out until they are no longer observable in the nebulosity.

Of particular interest are the polarization measurements which have been made in the optical. A rather substantial net polarization exists ($\sim$ 10%), and on a scale size of $\sim$ 1/10 of the size of the object (a few arc min), the polarization can exceed 50%. In fact, the polarization appears to be quite well-ordered, as a snapshot of the Crab through a polaroid filter yields a remarkably regular basket-weave pattern. The optical spectrum is well represented by a power law, as is the radio spectrum (albeit with different index). Because of this substantial polarization, and the fact that the optical and radio spectra are well represented by power laws (which we believe are indicative of non-thermal processes), a synchrotron origin for both of these emission bands is universally accepted.

The x-ray observations which have been made to date have been such that all of the data from ~ 1/4 keV to ~ 1/4 MeV are completely consistent with a structureless power law of index -2 (in the differential photon spectrum), which, when extrapolated down to optical frequencies, is in good agreement with the optical measurements[17]. Given that the optical and radio emission must be synchrotron in origin (owing to the observed polarization), the electron spectrum in the nebula can be constrained in order that an evaluation of the possible x-ray production mechanisms be made. We shall utilize the fact that the observed differential photon spectral index in the radio region is ~ -1.3, and that in the optical and x-ray region is ~ -2.0, with the break occurring at ~ 10^{14} Hz. We remark that the radio, optical and x-ray continua appear to be spread over the whole nebulosity, with any localized contribution being $\lesssim$ 10%. The break frequency prescribes a break in the radiating electron spectrum at:

$$\gamma^2 \approx \frac{10^8}{H_\perp} \approx 10^{11} \tag{69}$$

for a nebular field of ~ 10^{-3} gauss. It remains to show that for this electron spectrum, the synchrotron emission in the x-ray domain overcomes the Compton-produced x-rays from the electrons at lower energies in order that the model remain consistent. We must have:

$$\begin{aligned} \frac{d}{d\gamma}\frac{dN_1}{dV} &\propto \gamma^{-\Gamma_1} & \gamma &\lesssim 3\times10^5 \\ \frac{d}{d\gamma}\frac{dN_2}{dV} &\propto \gamma^{-\Gamma_2} & \gamma &\gtrsim 3\times10^5 \end{aligned} \tag{70}$$

continuous at $\gamma = 3 \times 10^5$, where

$$\begin{aligned} \frac{\Gamma_1+1}{2} &= 1.3 & \Gamma_1 &= 1.6 \\ \frac{\Gamma_2+1}{2} &= 2.0 & \Gamma_2 &= 3 \\ \frac{dN_1}{dN_2} &= 1 = C\frac{\gamma^{-1.6}}{\gamma^{-3}} & C &= 2\times10^{-8} \end{aligned} \tag{71}$$

Remembering that in such a nebula, we expect the starlight density to be $\sim 10^2$ eV/cm^3, $\gamma_c^2 \approx 4 \times 10^3$ and $\gamma_s^2 \approx 2 \times 10^{15}$, the Compton-to-synchrotron-produced x-ray ratio at 10 keV will be:

$$\mathcal{R} = \frac{8\pi\rho}{H^2}\left(\frac{\gamma_c}{\gamma_s}\right)^2 C\, \frac{\gamma_s^{3}}{\gamma_a^{1.6}} \approx 10^{-2} \tag{72}$$

so that synchrotron emission is clearly dominant. At 100 keV, the ratio is only 10^{-1}, so that at still higher energies, we expect synchrotron emission to cease being the dominant mechanism.

The radiative lifetime of these electrons is extremely short. For the x-ray producing electrons, $\tau \approx 1$ year, while even for the electrons around the break in the infra-red the lifetime is only ~ 100 years, still an order of magnitude less than the lifetime of the nebula. It is clear that any synchrotron model must be capable of replenishing the electrons which we assume are responsible for the x-ray emission in a time which is short compared to a year.

The difficulty involved in both the production and sustenance of such electrons led some observers to assume a thermal nature for the Crab, where non-relativistic electrons are involved of a different population than those responsible for the radio and optical emission. There are several observational problems with this alternative approach. Firstly, the observed spectrum over three orders of magnitude in x-rays is well-represented by a power law, which would argue against thermal bremsstrahlung unless the temperature gradient was continuous in the source in just such a way as to masquerade as a power law. Non-thermal bremsstrahlung would involve some replenishing of the electrons, but their lifetime would be considerably longer than that of the synchrotron-

producing electrons. More serious difficulties with a bremsstrahlung model are the absence of prominent emission lines due to the collisional excitation of the nebular gas, and the fact that the total nebular emission involves a considerably larger mass than there exists in the nebula.

There remain, therefore, two pressing problems with regard to the Crab. To absolutely clinch the synchrotron hypothesis, we want to measure polarization. To date, the experimental picture is such that an upper limit of $\sim 20\%$ on the net polarization exists[18]. Second, we must find a way to replenish the ultra-relativistic electrons in a time short compared to a year. Even the wisps have too small a frequency as experiments over the last half-dozen years have indicated a constancy in the Crab output to much better than 10%, and the wisps average about 3 months between appearances. The answer to the second of these problems, the origin of the ultra-relativistic electrons, appears to lie in the Crab pulsar, NP0531.

The pulsar was discovered in the radio in 1968[19], and its identification made with the same optical counterpart which Baade had indicated was the probable source of the wisps almost 30 years before. The pulsation frequency is about 30 cps, and the current belief is that the object is, in fact, a neutron star.

We believe that NP0531 (and all other pulsars) is a rotating object, because all pulsars, without exception, seem to be slowing down with age. The 30 msec period puts an absolute upper limit on the radius of the object such that its periphery can travel no faster than the speed of light:

$$R = \frac{cP}{2\pi} \approx 10^8 \text{ cm} \tag{73}$$

This is approximately the radius of a white dwarf, but one which spins this fast cannot possibly remain intact. While 10^8 cm is an approximate upper limit for the radius of the object, we can get a crude lower limit from:

$$G\frac{M}{R^2} \approx \frac{c^2}{R} \tag{74}$$

For $M \approx M_O$, $R \approx 10^5$cm is that radius for which particles on the surface will no longer be gravitationally bound. A neutron star, with radius $\sim 10^6$ cm and mass $\sim 1\ M_O$ is the only stable object known in this domain. While the above arguments are grossly simplified, the rough estimates obtained are sufficient for our purposes.

Neutron stars had been postulated as a possible remnant star in a supernova thirty years earlier, but had never been observed. Such objects undergo nuclear decomposition during the gravitational collapse, and the Fermi pressure arising from the Pauli principle halts the collapse at nuclear densities. In order to estimate some neutron stellar parameters, consider an ordinary stellar object, i.e., $\sim 1\ M_\odot$, $\sim 1 R_\odot = 10^{11}$ cm and with a solar rotation period $P_o \sim 2 \times 10^6$ sec. If we shrink the object to the size of a neutron star while conserving angular momentum:

$$\frac{R_\odot^2}{P_\odot} = \frac{10^{12}}{P}, \quad P = 10^{-4} \text{ sec} \tag{75}$$

This is about two orders of magnitude less than the smallest observed period, but is not unreasonable in view of the fact that we have neglected any loss of angular momentum in the supernova.

If we assume that the star has a dipole field of 100 gauss, conservation of flux will yield

$$H = \left(\frac{10^{11}}{10^{6}}\right)^2 \times 10^2 = 10^{12} \text{ gauss} \tag{76}$$

If the dipole axis is at all offset from the axis of rotation, magnetic dipole radiation at the rotation frequency will be emitted, with the power in this mode proportional to the projection of the dipole on the equatorial plane. The parallel component will result in the loss of rotational kinetic energy via the production of electric fields along the polar axis. Regardless of its inclination, the loss of rotational kinetic energy in electromagnetic modes is:

$$-\frac{dW}{dt} \propto \frac{B_0^2}{P^4} \tag{77}$$

where B_0 is the surface field. There is also energy loss via gravitational quadrupole radiation, but, as it is proportional to P^{-6}, it will become less important as the period increases[20]. Furthermore, as such quadrupole radiation will escape the nebula, it will play no role in the overall <u>nebular</u> energy balance.

We can form some simple conclusions about the temporal history of the object if we assume that the surface field does not decay.

$$-\frac{dW}{dt} = \frac{d}{dt}\left(\frac{1}{2}I\omega^2\right) = A B_0^2/P^4$$
$$\therefore B_0^2 = \frac{4\pi^2}{A} I P\dot{P} \tag{78}$$

We can then deduce a period-age relation:

$$\frac{dP}{dt} = \dot{P} \qquad \frac{P^2}{2} - \frac{P_0^2}{2} = (P\dot{P})t$$
$$P^2 = \frac{A}{2\pi^2 I} B_0^2 t - P_0^2 \tag{79}$$

and, in the limit $P >> P_0$:

$$t = \frac{2\pi^2 P^2 I}{A B_0^2} = \frac{1}{2}\frac{P}{\dot{P}} \qquad (80)$$

So that the age is directly measurable in terms of observables (P and $\dot{P}$), as is the surface magnetic field since the constant A should not vary substantially from pulsar to pulsar, in view of the fact that the masses and radii of all neutron stars are approximately the same.

Using 1 M_0 for the Crab, $R = 10^6$ cm and the simplified assumption of a uniform density:

$$I \approx \frac{3}{5} M R^2 = 10^{45} \text{ g-cm}^2 \qquad (81)$$

For the Crab, equation (80) and the known age gives

$$\dot{P} = \frac{1}{2}\frac{P}{t} \approx \frac{1}{2} \times 10^{-12} \qquad (82)$$

which is not very different than the measured value, as a consistency check, so that we can evaluate the kinetic energy loss rate:

$$-\frac{dU}{dt} = 4\pi^2 I \frac{\dot{P}}{P^3} \approx 10^{39} \text{ ergs/sec} \qquad (83)$$

The observed photon spectrum from the Crab Nebula is

$$\frac{dF}{d h\nu} \approx 8 (h\nu)^{-2} \text{ cm}^{-2}\text{sec}^{-1}\text{keV}^{-1} \qquad (84)$$

the "accepted" distance to the Crab is ~ 1.5 kpc, so that the net output of the Crab between two photon energies is:

$$4\pi d^2 \int_{E_1}^{E_2} 8 \frac{h\nu \, d(h\nu)}{(h\nu)^2} \approx 5 \times 10^{36} \ln \frac{E_2}{E_1} \qquad (85)$$

or $\sim 10^{37}$ ergs/sec per decade of energy, so that the loss of

rotational kinetic energy can, on an energetics basis, easily account for the output of the nebula in all observed energy bands.

The problem of continuous energy supply to the nebula is then solved, or perhaps we should say amenable to solution. For, although the solution is qualitatively understandable, the pulsar electrodynamics are by no means well-understood. The x-ray emission exhibits the same pulsed nature as does the radio, with a much higher pulsed-to-non-pulsed fraction (i.e., 10^{-1}). In fact, the pulsed fraction appears to be continually increasing as a function of energy even over the limited range of x-ray data available[21].

The Crab, as attractive a prototype for x-ray emitting supernova as it is, has many particularly anomalous characteristics. It is the only x-ray source from which pulsed emission has been observed, even though both Cas A and Tycho are younger supernovae and are, likewise, strong x-ray emitters. It is also the highest frequency pulsar in the whole pulsar catalog. In the simplified treatment of pulsar slowing-down theory we have neglected the possibility of magnetic decay, which many theorists believe is necessary to account for all features of the pulsar sample[22,23]. Although this plays no role for an object as young as the Crab, the great bulk of the pulsar sample is in the age bracket where it should be quite important ($\sim 10^6$ years). There are also many differences between the Crab and most other remnants in the radio and optical[24]. In studying x-ray sources, supernovae and pulsars then, great care must be taken to avoid using the Crab as a prototype, as the differences between the Crab and comparison candidates often outnumber the similarities.

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